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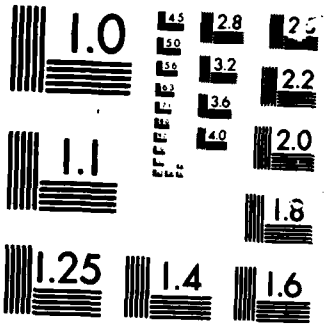

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During the past 7 months, the research efforts supported by this grant have concentrated on robust estimation techniques for autoregressive models and some related system theoretic problems associated with parameter estimation problems for time series models. The motivation for this work arises from applicaitons in signal processing such as linear predictive signal modeling, signal detection, and spectral analysis. The overall goal of this research has been put together ideas and techniques from statistics, signal processing, and system theory to bring new perspectives to such problems.

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Research Progress and Forecast Report

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH GRANT AFOSR-84-0381

Efficient Algorithms and Structures for Robust Signal Processing

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Report of Research Progress

During the past 7 months, the research efforts supported by AFOSR Grant AFOSR-84-0381 have concentrated on robust estimation techniques for autoregressive models and some related system theoretic problems associated with parameter estimation problems for time series models. The motivation for this work arises from applications in signal processing such as linear predictive signal modeling, signal detection, and spectral analysis. The overall goal of this research has been to put together ideas and techniques from statistics, signal processing, and system theory to bring new perspectives to such problems. This section will be devoted to some discussion of the technical results obtained to date.

Our research on autoregressive modeling has been motivated by the need to relax some of the unrealistic assumptions made by previous researchers, allowing for robust estimation in the presence of outliers in the observed signal. We have made extensive

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use of a particular model structure, the so-called lattice structure, to reduce the computational complexity of the iterated, reweighted least squares approach to outlier suppression [1]. The structure uses partial autocorrelations to parametrize the autoregression, resulting in a decoupling of the least squares problems. Results in [1] show that just one or two iterations of the basic "data cleaning" procedure provides estimates with much better performance than least squares.

Given the success of this approach to robust autoregressive model fitting, we have begun to explore how our approach may be extended to more general signal modeling and analysis problems. As a first step, we examined how the lattice structure may be used to extract estimates of the AR part of an ARMA model [2]. Such a problem arises when applying linear prediction (autoregressive modeling) for spectral estimation of sine waves in additive white noise. We have shown that it is not necessary to solve a sequence of least squares problems (of order greater than or equal to the autoregressive order) to get consistent estimates; rather an extended lattice filter may be transformed to give the estimates from the solution of a single set of linear equations (of order equal to the moving average order) [2]. This result is also shown to be equivalent to the extended Yule-Walker approach. The principal advantage will come when we apply the robust estimation techniques already mentioned to this more general estimation problem.

We turn now to a second area of research, starting with some notation for the kind of problems studied. Let $(U_1, U_2, \cdots)$ be a sequence of observed random variables whose probability distributions are described by a parametrized family of density functions $\{p_k(u_1, \cdots, u_k; \theta)\}$. We assume that there exists a sequence of sufficient statistics for θ, $(T_1(U_1), T_2(U_1, U_2), \cdots)$, so that all of the information about the parameter θ available in the observations up to instant k is preserved in the (fixed dimension!) statistic T_k. This formulation is applicable to many parameter estimation problems arising in applications, including linear predictive (i.e. autoregressive) modeling of signals.

In the study of problems where the observation record increases, it is a natural system theoretic view to regard a sequence of sufficient statistics as defining the input/output map of a dynamical system. In work completed during the current grant

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period, done in collaboration with Eduardo Sontag of Rutgers University, we have developed some realization theory for nonlinear, time varying systems that can be applied to the study of state space models for sufficient sequences [3]. The importance of state space models should be clear: they provide a recursive means of updating the sequence of sufficient statistics as more observations are made. Actually, for this to be the case, it is necessary that the state space models be finite dimensional. Results in [3] indicate classes of systems for which finite dimensional realizations may be obtained and give a criterion for verifying that a (nonlinear, time varying) state space model is a minimum dimension realization of its input/output map. The latter is an appropriately formulated theorem of the usual kind: controllability and observability of a realization imply that the realization is minimum dimension.

In [3] we adopted the framework of nonlinear filtering theory, i.e. the Bayesian perspective, and proved that $3p+1$ is the minimum dimension realization of the recursive Bayes estimator for the parameters of a pth order autoregressive process. This estimator is based on reproducing (or conjugate) families of densities, but it is equivalent, at least for long observation records, to the usual least squares estimates used in linear predictive modeling.

Another subject considered in [3] is the relation of the notion of realizability of a sufficient sequence with other properties introduced in the statistics literature, especially transitivity. We show that realizability has some important practical implications through its connection with (finite dimensional) recursive equations for updating the sequence of statistics that are not necessarily associated with transitivity.

We have followed up our work on realizability with some further study of its implications and limitations. The theory in [3] requires that realizations take the form

$$X_{k+1} = \sigma_k(X_k, U_k)$$

$$T_k = \eta_k(X_k, U_k)$$

where the functions σ and η are differentiable, since we need some kind of smoothness condition to make sense out of the notion of dimension. In [3] we gave the following

example to show that there are sufficient sequences admitting no smooth finite dimensional realization. For observations from a stationary Gaussian process with unknown mean and known, nonrational power spectral density function, there is a one-dimensional sufficient sequence defining a linear input/output map that has no smooth finite dimensional realization, linear or nonlinear. Viewed from the perspective of the observed process, this result is perhaps not unexpected, since there is no smooth finite dimensional *model which generates* such observations.

In following up on this example, we have now shown [4] that its essential feature, no finite dimensional model for the observation process, must underlie any example of this sort. Specifically, a realizability assumption, in conjunction with existence of a sufficient sequence, implies that the observation process is equivalent to one that is generated by a finite dimensional dynamical system driven by a sequence of independent random variables.

In further work, still in progress but reported in summary form in [5], we have extended our approach to deal with the problem of prediction (where the parameters of the joint probability density function of the observations are of no interest). In practice, prediction problems usually arise in connection with a growing observation record instead of a fixed one. As described above, if there exists a sequence of sufficient statistics for the parameter, as in the case of an autoregressive process, then the sequence may be viewed as the input-output map of a dynamical system and the question of finite dimensional realizability may be investigated. For prediction problems, there are interesting relationships between realizability and a property called total sufficiency. For autoregressive processes, For autoregressive processes, this property holds and provides one way of obtaining parameter-independent predictions.

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Research Plans

We turn now to a brief overview of research planned for the second year of the grant period. As indicated above, our research in robust signal modeling is ongoing, with current efforts directed along two lines. First, extending the robust AR estimation method to more general models where the lattice filter structure can be exploited will be considered in detail. Second, we hope to merge the two lines of research described above through the work on sufficiency and prediction. Specifically, using nominal forms for the predictors obtained for certain parametric models (such as autoregressions) we plan to explore procedures for making these predictors robust.

There are several topics associated with realizability of sufficient sequences that remain for further investigation. We plan to explore the implications of realizability of a sufficient sequence on the structure of the family of joint density functions for the

observed process. It appears that our work should lead to a significant generalization of the well-known result for sequences of independent observations, namely that realizability implies an exponential family form. The realizability assumption appears to be just the right one to treat sequences of dependent (correlated) random variables, which is the case of interest for signal processing applications.

Related to this work will be an investigation of how results on parametrized families of systems might be used to provide a useful definition of a parametrized family of joint density functions corresponding to a class of processes with "finite dimensional representations." What we have in mind here is a definition that will work when sufficiency doesn't help. For example, a filtered Gaussian white noise sequence has a finite dimensional representation whenever the filter transfer function is a rational function (as a z-transform), even though a sufficient statistic exists (and by the work described earlier, leads to a finite dimensional representation) only if the filter is all-pole. Removing the assumption of regularity of the family of joint density functions would also be of interest, allowing a wider class of models to be treated.

Further investigation of the connections between sufficiency and prediction for time series models will be carried out. As noted above, we hope to be able to use this approach as part of our work on robustness. We want to give a thorough answer to the question of why finite dimensional sufficient statistics and finite dimensional predictor spaces seem to go hand in hand. Because there is some well-developed theory of approximate (i.e. in the sense of least squares) prediction, we believe that the connections between prediction and sufficiency will lead to a reasonable theory of approximation in the context of sufficiency, applicable to signal analysis problems such as linear predictive modeling. Clearly, the whole question of approximation is an important one, being the essential part of the modeling problem in any real application.

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